Int J Theor Phys (2009) 48: 2112–2121
DOI 10.1007/s10773-009-9988-y

Hawking Radiation of Apparent Horizon in a FRW Universe as Tunneling Beyond Semiclassical Approximation

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Received: 3 December 2008 / Accepted: 9 March 2009 / Published online: 19 March 2009

Abstract An analysis of Hawking radiation about apparent horizon in a FRW universe is performed by using the method developed in the paper (Banerjee, Majhi in JHEP 06:095 2008), in which the Hawking radiation of a black hole is treated as the quantum tunneling by Hamilton-Jacobi method beyond semiclassical approximation and then all the higher order quantum corrections can be given out. In our analysis, the Kodama vector instead of the Killing vector to define the energy of the particle plays a key role. We present our analysis under the Friedmann-Robertson-Walker like coordinate system and the much-like to Painlevé coordinate system respectively. The result show that the formulized procedure can be extended to fully analyse the Hawking radiation of a dynamical system.

Keywords Black holes · Hawking radiation · Tunneling

1 Introduction

Black holes are so charming and important in modern theoretical physics and astrophysics. In particular, the discovery of Hawking radiation shows that black holes are not completely black [1, 2], but quantum mechanically, emit radiation with a thermal spectrum like a black body. Then we come to realize that black holes may play the role of "Rosetta stone" to related some of the main branches of physics theories: gravity, quantum theory and thermodynamics and enable us a to better understand these modern physics theories. Therefore, Hawking radiation, as one of the most striking effects that arise from the alliance of quantum theory and general relativity, has attracted widespread interest in physics community.

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The relationship between thermodynamics and Einstein equations have been given much attention, since the discovery of black hole thermodynamics [2–4]. In 1995, Jacobson [5] found that the Einstein equation can be viewed as nothing, but an equation of state. The Einstein equation can be derived from the fundamental equilibrium thermodynamics first law relation $\delta Q = TdS$ together with the proportionality of entropy and the horizon area, presuming that the relation holds for all Rindler causal horizons through each spacetime point. Another outstanding linking evidence comes from Verlinde's observation [6] in which he pointed out that the Friedmann equation of a radiation dominated Friedmann-Robertson-Walker (FRW) universe is exactly a cosmological Cardy formula, which has the same form with the conformal field theory (CFT) entropy formula in a higher dimensional spacetime, the Cardy-Verlinde formula. All of these indicate that the dynamical equation which describes spacetime has the same form with the entropy formula of CFT's which describes the thermodynamical of radiation in the universe. Obviously, Verlinde's discovery further indicates the relationship between thermodynamics and Einstein equations.

In the spirit of Jacobson's approach, by assuming the apparent horizon has a temperature $T = 1/(2\pi\tilde{r})$ and entropy $S = A/4$, with $\tilde{r}$ and A are the radius and the area of the apparent horizon respectively, [7, 8] showed that the Friedmann equations of a FRW universe can be derived from the first law of thermodynamics and this only exactly holds for apparent horizon in the FRW universe. The relationship between thermodynamics and Einstein equations have been explored by many authors in literature (for incomplete references see [8–13]). This problem also has been discussed in brane world scenarios, see [14] and references there in. But, is there indeed a Hawking radiation with temperature $T = 1/(2\pi\tilde{r})$ associated with the apparent horizon of the FRW universe?

Since Hawking's original work [1, 2] in 1970's, the methods about the radiation of black holes were completely based on quantum field theory. Several other derivations of Hawking radiation were subsequently presented in the literature [15–17]. However, none of these methods corresponds very directly to one of the heuristic pictures most commonly proposed to visualize the source of the radiation, as tunneling. In 2000, to overcome this shortcoming, Parikh and Wilczek [18] proposed a tunneling method in which the Hawking radiation was presented as a tunneling process, based on particles in a dynamical geometry, and the imaginary part of the action for the classically forbidden process is related to Boltzmann factor of emission at the Hawking temperature. There are two different methods to calculate the imaginary part of the action: one is developed by Srinivasan et al. [19]—the Hamilton-Jacobi method, another is present by Parikh and Wilczek [18]—radial null geodesic method. Note that all these computations are confined to semiclassically only. Quantum correction is generally not included in. Many works [20–38] have been investigated for further development of this approach, and the method worked perfectly. However, for criticism and counter criticism see [39–43].

Recently, a so-called Hamilton-Jacobi method of tunneling beyond semiclassical approximation have been formed [44–48] to analyse the Hawking radiation of black holes generally. One of the remarkable points of this approach is that the procedure can give out all possible quantum corrections. Through a simple choice of the proportionality constants, the one loop back reaction effect in the spacetime [49, 50], found by conformal theory methods [51], can also be obtained, which modifies the Hawking temperature. Despite of their formulated procedure is general enough, their approach mainly based on stationary system in which the timelike Killing vectors can be used to define the particle energy. Technically, Killing vectors play a crucial role in giving the form of the action solutions. But for the dynamical setting, the case has some differences. So it is interesting to apply their formulized procedure to analyse the dynamical system.

Interestingly, using the Parikh-Wilczek tunneling approach [18], Cai et al. [52], recently, finished the proof that the apparent horizon of FRW universe has indeed a Hawking radiation with temperature $T = 1/(2\pi\tilde{r})$. In addition, they pointed out that Hawking radiation is not always associated with event horizon of spacetime, in other words, the existence of event horizon is not always a key cause of Hawking radiation. In this paper, we will investigate this case. By extending the method formed in [44, 45] to a dynamical case, we will fully calculate the Hawking radiation of apparent horizon for a FRW universe.

By means of formulated approach developed in [44, 45] for stationary black holes, however, for the dynamical FRW universe setting, there are some interesting details of the solving procedure should be highlighten here, which are different from the stationary black holes case. In a dynamical setting, there is a preferred time direction given by the Kodama vector [53–56], which is a natural analogue to the Killing vector to define the energy of a particle. Related to this choice of time direction, we can give the form of action solutions in the dynamical case. As in [7] pointed that the existence of the Kodama vector play a crucial role in their discussion, we find that, by using the method of tunneling beyond semiclassical approximation for a dynamical system, the Kodama vector still play a crucial role. Another difference is, for black holes, Hawking radiation comes from positive energy outgoing particles, however, in the case of apparent horizon in a FRW universe, since the observer is inside the apparent horizon, we should consider the incoming modes instead of the outgoing modes [52].

There are two coordinate systems can be used to discuss the Hawking radiation of apparent horizon in a FRW universe, one is the FRW like coordinate system, the other is much like to Painlevé coordinate system. So we will present our procedure with these two different coordinate systems in Sects. 2 and 3 respectively. In the final section we give out our conclusions and a brief remark.

Throughout the paper, we use units with $G = c = k_B = 1$, but write down the Planck constant explicitly.

2 Friedmann-Robertson-Walker Like Coordinate System

In this section, we will analyse the Hawking radiation of apparent horizon in a FRW universe under the FRW like coordinate system. The analysis goes beyond the semiclassical approximation by including all possible quantum corrections. Here the procedure is parallel with the procedure in [44, 45].

For simplicity, let us start with the 4-dimensional FRW metric

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right), \tag{1}$$

where t represent the cosmic time, r represent the comoving coordinate, $a(t)$ represent the scale factor of the universe, $d\Omega^2$ refers to the line element of a 2-dimensional unit sphere, and $k = -1, 0, 1$ is the spatial curvature constant which corresponds to an open, flat and closed universe respectively. Conveniently, using spherical symmetry, the metric (1) can be written as

$$ds^2 = h_{ab}dx^a dx^b + \tilde{r}^2(t,r)d\Omega^2, \tag{2}$$

where $\tilde{r}(t,r) = a(t)r$. We will let the Greek indices $\alpha, \beta, \ldots$ running over the whole four coordinates, while the Latin indices $a, b, \ldots = 0, 1$ denote the cosmic time t and the comoving coordinate r respectively. The reduced 2-dimensional metric $h_{ab} = \mathrm{diag}(-1, a^2(t)/$

$(1-kr^2)$). The apparent horizon $\mathcal{H}$ for the metric (2) is defined as

$$h^{ab}(\nabla_a\tilde{r})(\nabla_b\tilde{r})|_{\mathcal{H}}=0, \tag{3}$$

where ∇_a denotes the covariant differential with respect to the metric (2). Using the metric (2) and (3), one can easily get the radius of the apparent horizon for the FRW universe

$$\tilde{r}_A=\frac{1}{\sqrt{H^2+k/a^2}}, \tag{4}$$

where $H\equiv\dot{a}/a$ is the Hubble parameter, the overdot stands for the derivative with respect to t.

Now, let us consider a massless particle in the spacetime (2) described by the Klein-Gordon equation

$$-\hbar^2\nabla^\mu\nabla_\mu\phi=0. \tag{5}$$

Since for the radial trajectories only the $(t-r)$ sector is important, for the reduced metric (2), from (5), we have

$$-\partial_t^2\phi-H\partial_t\phi+\left(\frac{a^2}{1-kr^2}\right)^{-1}\partial_r^2\phi-\frac{kr}{a^2}\partial_r\phi=0. \tag{6}$$

By making the standard ansatz for ϕ

$$\phi(t,r)=\exp\left[-\frac{i}{\hbar}S(t,r)\right], \tag{7}$$

where $S(t,r)$ is one particle action which will be expanded in powers of $\hbar$, the semiclassical wave functions satisfying (6) can be obtained. Substituting (7) into the wave equation (6), we obtain

$$(\partial_t S)^2-\left(\frac{a^2}{1-kr^2}\right)^{-1}\left(\partial_r S\right)^2-\left(\frac{\hbar}{i}\right)\left[\partial_t^2 S-\left(\frac{a^2}{1-kr^2}\right)^{-1}\partial_r^2 S+H\partial_t S+\frac{kr}{a^2}\partial_r S\right]=0. \tag{8}$$

Now, expanding S in a power series of $(\hbar/i)$ yields

$$S(t,r)=S_0(t,r)+\left(\frac{\hbar}{i}\right)S_1(t,r)+\left(\frac{\hbar}{i}\right)^2 S_2(t,r)+\cdots. \tag{9}$$

With this expanding, by equating the different powers of $\hbar$ on both sides, and inserting the lower order to the next higher, after a straightforward calculation, (8) gives us

$$\begin{aligned}
\left(\frac{\hbar}{i}\right)^0 &: \partial_t S_0=\pm\sqrt{(1-kr^2)/a^2}\,\partial_r S_0,\\
\left(\frac{\hbar}{i}\right)^1 &: \partial_t S_1=\pm\sqrt{(1-kr^2)/a^2}\,\partial_r S_1,\\
\left(\frac{\hbar}{i}\right)^2 &: \partial_t S_2=\pm\sqrt{(1-kr^2)/a^2}\,\partial_r S_2,\\
&\vdots
\end{aligned} \tag{10}$$

We see that the liner differential equations for each order of the $(\hbar/i)$ have the same form. Obviously, the solutions of these equations are not independent and all the S_i's, where $i = 1, 2, \ldots$, are proportional to S_0. The proportionality constants have the dimension of $(1/\hbar)^i$ is obviously since S_0 has the dimension of $\hbar$. In our units conversion, the Planck constant $\hbar$ has the order of $(M_P)^2$ where M_P is the Planck Mass. So the proportionality constants have the dimension of $(1/E^2)^i$, where $E = \tilde{r}_A/2$ is the total energy inside the spherical radius of apparent horizon (4) defined in [8]. Now following (9), the most general expression of S can be expressed by

$$\begin{aligned} S(t,r) &= S_0(t,r) + \beta_1 \frac{\hbar}{E^2} S_0(t,r) + \beta_2 \left(\frac{\hbar}{E^2}\right)^2 S_0(t,r) + \cdots \\ &= S_0(t,r)\left[1 + \beta_1 \frac{\hbar}{E^2} + \beta_2 \left(\frac{\hbar}{E^2}\right)^2 + \cdots\right], \end{aligned} \tag{11}$$

where β_i's are dimensionless constant parameters.

As we have highlighten in the first section, now, we introduce the Kodama vector. For the metric (2), the Kodama vector is

$$K^a = -\epsilon^{ab}\nabla_b \tilde{r} = -\sqrt{1-kr^2}[-(\partial_t)^a + Hr(\partial_r)^a], \tag{12}$$

where $\epsilon_{ab} = a(t)/\sqrt{1-kr^2}(dt)_a \wedge (dr)_b$. Using the Kodama vector, one can define the energy of the particle ω as,

$$\omega = -K^a \nabla_a S = \sqrt{1-kr^2}(-\partial_t S + Hr\partial_r S). \tag{13}$$

Inserting the expression of S (11) into (13), we can obtain the equation satisfied by the lowest order of S

$$\omega = -K^a \nabla_a S_0 = \sqrt{1-kr^2}(-\partial_t S_0 + Hr\partial_r S_0). \tag{14}$$

From the first equation of (10) and (14), we have

$$\partial_t S_0 = \frac{\omega(\mp H\tilde{r} - \sqrt{1-k\tilde{r}^2/a^2})}{1-\tilde{r}^2/\tilde{r}_A^2}, \tag{15}$$

$$\partial_r S_0 = \frac{a\omega(-H\tilde{r} \mp \sqrt{1-k\tilde{r}^2/a^2})}{(1-\tilde{r}^2/\tilde{r}_A^2)\sqrt{1-k\tilde{r}^2/a^2}}, \tag{16}$$

where the $+(-)$ sign indicates the particle is outgoing (ingoing). Rewrite S_0 as

$$S_0(t,r) = \int dt \frac{\partial S_0}{\partial t} + \int dr \frac{\partial S_0}{\partial r}, \tag{17}$$

and combine (15), (16), (17) and (11), we obtain

$$\begin{aligned} S(t,r) &= \left[-\int dt \frac{\omega}{\sqrt{1-k\tilde{r}^2/a^2}} + \int d\tilde{r} \frac{\omega(-H\tilde{r} \mp \sqrt{1-k\tilde{r}^2/a^2})}{(1-\tilde{r}^2/\tilde{r}_A^2)\sqrt{1-k\tilde{r}^2/a^2}}\right] \\ &\quad \times \left[1 + \beta_1 \frac{\hbar}{E^2} + \beta_2 \left(\frac{\hbar}{E^2}\right)^2 + \cdots\right], \end{aligned} \tag{18}$$

here, we have used the differential relation $d\tilde{r} = adr + H\tilde{r}dt$ to replace the variable r with $\tilde{r}$.

Using (7), we get the ingoing and outgoing solutions of the Klein-Gordon equation

$$\phi_{\text{in}} = \exp\left\{-\frac{i}{\hbar}\left[-\int \frac{dt\omega}{\sqrt{1-k\tilde{r}^2/a^2}} + \int d\tilde{r}\frac{\omega(-H\tilde{r} - \sqrt{1-k\tilde{r}^2/a^2})}{(1-\tilde{r}^2/\tilde{r}_A^2)\sqrt{1-k\tilde{r}^2/a^2}}\right]\right.$$
$$\left.\times\left[1+\beta_1\frac{\hbar}{E^2}+\beta_2\left(\frac{\hbar}{E^2}\right)^2+\cdots\right]\right\}, \tag{19}$$

$$\phi_{\text{out}} = \exp\left\{-\frac{i}{\hbar}\left[-\int \frac{dt\omega}{\sqrt{1-k\tilde{r}^2/a^2}} + \int d\tilde{r}\frac{\omega(-H\tilde{r} + \sqrt{1-k\tilde{r}^2/a^2})}{(1-\tilde{r}^2/\tilde{r}_A^2)\sqrt{1-k\tilde{r}^2/a^2}}\right]\right.$$
$$\left.\times\left[1+\beta_1\frac{\hbar}{E^2}+\beta_2\left(\frac{\hbar}{E^2}\right)^2+\cdots\right]\right\}. \tag{20}$$

The essence of tunneling based calculation is the computation of the imaginary part of the action of a system for the (classically forbidden) process of s-wave emission across the horizon, which in turn is related to the Boltzmann factor for the emission at the Hawking temperature. Therefor, following the standard approach, the ingoing and outgoing probabilities of the particle can be given by

$$P_{\text{in}} = |\phi_{\text{in}}|^2$$
$$= \exp\left\{\frac{2}{\hbar}\left[-\text{Im}\int \frac{dt\omega}{\sqrt{1-k\tilde{r}^2/a^2}} + \text{Im}\int d\tilde{r}\frac{\omega(-H\tilde{r} - \sqrt{1-k\tilde{r}^2/a^2})}{(1-\tilde{r}^2/\tilde{r}_A^2)\sqrt{1-k\tilde{r}^2/a^2}}\right]\right.$$
$$\left.\times\left[1+\beta_1\frac{\hbar}{E^2}+\beta_2\left(\frac{\hbar}{E^2}\right)^2+\cdots\right]\right\}, \tag{21}$$

$$P_{\text{out}} = |\phi_{\text{out}}|^2$$
$$= \exp\left\{\frac{2}{\hbar}\left[-\text{Im}\int \frac{dt\omega}{\sqrt{1-k\tilde{r}^2/a^2}} + \text{Im}\int d\tilde{r}\frac{\omega(-H\tilde{r} + \sqrt{1-k\tilde{r}^2/a^2})}{(1-\tilde{r}^2/\tilde{r}_A^2)\sqrt{1-k\tilde{r}^2/a^2}}\right]\right.$$
$$\times\left[1+\beta_1\frac{\hbar}{E^2}+\beta_2\left(\frac{\hbar}{E^2}\right)^2+\cdots\right]. \tag{22}$$

As we have highlighten in Sect. 1, since the observer is inside the apparent horizon of FRW universe, we should consider the incoming mode, while the outgoing probability P_{out} has to be unity in the classical limit (i.e. $\hbar \to 0$). Thus from (22), in the classical limit we have

$$\text{Im}\int \frac{dt\omega}{\sqrt{1-k\tilde{r}^2/a^2}} = \text{Im}\int d\tilde{r}\frac{\omega(-H\tilde{r} + \sqrt{1-k\tilde{r}^2/a^2})}{(1-\tilde{r}^2/\tilde{r}_A^2)\sqrt{1-k\tilde{r}^2/a^2}}. \tag{23}$$

Inserting (23) into (21), we obtain the probability of the ingoing particle

$$P_{\text{in}} = \exp\left\{-\frac{4}{\hbar}\left[\text{Im}\int \frac{d\tilde{r}\omega}{(1-\tilde{r}^2/\tilde{r}_A^2)}\right]\left[1+\beta_1\frac{\hbar}{E^2}+\beta_2\left(\frac{\hbar}{E^2}\right)^2+\cdots\right]\right\}. \tag{24}$$

Now, we can use the principle of "detailed balance" [15, 19, 57]. In the FRW universe case, this principle can be expressed as follows

$$P_{\text{in}} = \exp\left(\frac{\omega}{T_h}\right) P_{\text{out}} = \exp\left(\frac{\omega}{T_h}\right), \tag{25}$$

in the same spirit in tunneling approach by [18] that the Hawking radiation of black hole which be expressed as a tunneling phenomenon [52].

Obviously, the integral in (24) has a pole at the apparent horizon $\tilde{r} = \tilde{r}_A$. From (24) and (25), by a contour integral, we have

$$T_h = T_H \left[1 + \beta_1 \frac{\hbar}{E^2} + \beta_2 \left(\frac{\hbar}{E^2}\right)^2 + \cdots\right]^{-1}, \tag{26}$$

where

$$T_H = \frac{\hbar}{2\pi \tilde{r}_A} \tag{27}$$

is the Hawking temperature corresponding to the apparent horizon in a FRW universe, which is in agreement with the arguments in [52], and the other terms can be explained as the corrections due to the quantum effects.

Indeed, we have shown that we can extend Banerjee and Majhi's method to a dynamical system, as an example, we analysed the so called Hawking temperature of apparent horizon in a FRW universe.

3 Much-Like to Painlevé Coordinate System

In this section, we will see that the method is also valid in a much-like to Painlevé coordinate system in the dynamical FRW universe case.

Considering the coordinate system $(t, \tilde{r}, \theta, \varphi)$, the metric (1) can be written as

$$ds^2 = -\frac{1 - \tilde{r}^2/\tilde{r}_A^2}{1 - k\tilde{r}^2/a^2} dt^2 - \frac{2H\tilde{r}}{1 - k\tilde{r}^2/a^2} dt d\tilde{r} + \frac{1}{1 - k\tilde{r}^2/a^2} d\tilde{r}^2 + \tilde{r}^2 d\Omega^2, \tag{28}$$

this coordinate is much like to the Painlevé coordinate, especially, when $k = 0$ the coordinate singularity at the horizon is removed. Conveniently, the metric (28) can be written as

$$ds^2 = \bar{h}_{ab} dx^a dx^b + \tilde{r}^2 d\Omega^2, \tag{29}$$

where the Latin indices $a, b = 0, 1$ denote t and $\tilde{r}$ respectively. For a massless particle, the Klein-Gordon equation (5), now, becomes

$$\begin{aligned} -\partial_t^2 \phi - \frac{H}{1 - k\tilde{r}^2/a^2} \partial_t \phi - 2rH \partial_t \partial_{\tilde{r}} \phi + \left(1 - \frac{\tilde{r}^2}{\tilde{r}_A^2}\right) \partial_{\tilde{r}}^2 \phi \\ - \frac{\tilde{r}}{\tilde{r}_A^2} \left(1 + \frac{1 - k\tilde{r}_A^2/a^2}{1 - k\tilde{r}^2/a^2}\right) \partial_{\tilde{r}} \phi = 0. \end{aligned} \tag{30}$$

Using (7), we obtain

$$(\partial_t S)^2 + 2\tilde{r} H \partial_t S \partial_{\tilde{r}} S - \left(1 - \frac{\tilde{r}^2}{\tilde{r}_A^2}\right)\left(\partial_{\tilde{r}} S\right)^2 - \left(\frac{\hbar}{i}\right)\left[\partial_t^2 S + \frac{H}{1 - k\tilde{r}^2/a^2}\partial_t S + 2\tilde{r} H \partial_t \partial_{\tilde{r}} S \right.$$
$$\left. - \left(1 - \frac{\tilde{r}^2}{\tilde{r}_A^2}\right)\partial_{\tilde{r}}^2 S + \frac{\tilde{r}}{\tilde{r}_A^2}\left(1 + \frac{1 - k\tilde{r}_A^2/a^2}{1 - k\tilde{r}^2/a^2}\right)\partial_{\tilde{r}} S\right] = 0. \tag{31}$$

Substituting (9) into (31), after a straightforward calculation and simplification, which the process similar with in the Sect. 2, we find

$$\begin{aligned}
\left(\frac{\hbar}{i}\right)^0 &: \partial_t S_0 = \left(-\tilde{r} H \pm \sqrt{1 - k\tilde{r}^2/a^2}\right)\partial_{\tilde{r}} S_0, \\
\left(\frac{\hbar}{i}\right)^1 &: \partial_t S_1 = \left(-\tilde{r} H \pm \sqrt{1 - k\tilde{r}^2/a^2}\right)\partial_{\tilde{r}} S_1, \\
\left(\frac{\hbar}{i}\right)^2 &: \partial_t S_2 = \left(-\tilde{r} H \pm \sqrt{1 - k\tilde{r}^2/a^2}\right)\partial_{\tilde{r}} S_2, \\
&\vdots
\end{aligned} \tag{32}$$

From the form of above series equations, based on the analysis in the Sect. 2, we can believe that the relation between S and S_0 in the so called much-like Painlevé coordinate system still satisfies (11). We don't write it out here, repeatedly.

Now, using the Kodama vector corresponding to the metric (29), the energy ω of the massless particle can be redefined as

$$\omega = -K^a \nabla_a S_0 = -\sqrt{1 - k\tilde{r}^2/a^2}\,\partial_t S_0. \tag{33}$$

From (17), using the first equation of (32) and (33), we have

$$S_0(t, \tilde{r}) = -\int dt \frac{\omega}{\sqrt{1 - k\tilde{r}^2/a^2}} + \int d\tilde{r} \frac{\omega(-H\tilde{r} \mp \sqrt{1 - k\tilde{r}^2/a^2})}{(1 - \tilde{r}^2/\tilde{r}_A^2)\sqrt{1 - k\tilde{r}^2/a^2}}. \tag{34}$$

Substituting (34) into (11) yields

$$S(t, \tilde{r}) = \left[-\int dt \frac{\omega}{\sqrt{1 - k\tilde{r}^2/a^2}} + \int d\tilde{r} \frac{\omega(-H\tilde{r} \mp \sqrt{1 - k\tilde{r}^2/a^2})}{(1 - \tilde{r}^2/\tilde{r}_A^2)\sqrt{1 - k\tilde{r}^2/a^2}}\right]$$
$$\times \left[1 + \beta_1 \frac{\hbar}{E^2} + \beta_2 \left(\frac{\hbar}{E^2}\right)^2 + \cdots\right], \tag{35}$$

which is identity with the expression (19).

The following steps are totally identity with in the Sect. 2 from (19) to (26). So we don't repeat them here. Now, we convince that, using the metric (28), one can once again reach the result.

4 Conclusion and Remarks

In conclusion, we have shown that, by means of the Kodama vector instead of the Killing vector to define the energy of the particle, the formulized method developed by Banerjee and Majhi to treat stationary black holes can be applied to a dynamical system. We find that the Kodama vector plays a crucial role, and this is no surprise, since the Kodama vector is a natural candidate to replace the Killing vector of a dynamic system [58]. As an example, we use the procedure to calculate the Hawking radiation of apparent horizon in a FRW universe. We give the Hawking temperature of apparent horizon in a FRW universe, which included all quantum corrections in a single particle action. The lowest order term corresponding to the standard semiclassical Hawking temperature, which is agree with in literature [52]. On the other hand, our calculation, further indicates that the apparent horizon of a FRW universe indeed assign a Hawking temperature.

For black hole case, through a simple choice of the proportionality constants β_i's, Banerjee and Majhi obtained the one loop back reaction effects in the spacetime or those based on conformal field theory techniques. And they also discussed the nature of two loop corrections to the surface gravity of the black hole. The similar treatment indicates that, in a FRW universe case, the proportionality constants β_i's (11) should also correspond to back reaction effects. However, this is not a priori conclusion. Since there is indeed a Hawking radiation associated with the apparent horizon of the FRW universe, further investigation of the exact thermal spectrum, which back reaction effects are taken into account, is needed.

Acknowledgements One of the authors (Kexia Chiang) is grateful to S.-M. Ke for useful discussion and kind help. This work was partially supported by the natural science research foundation of Shaanxi education bureau of China: 07JK394. T. Feng is supported by NWU Graduate Cross-discipline Funds (08YJC24).

Note Added in Proof While completing this work, we came across Ref. [59], which also reaches the same conclusion regarding the tunneling method beyond semiclassical approximation of black holes to the case of FRW universe. In fact unlike [59] only considering the much like to Painlevé coordinate system, we used both the FRW like coordinate system and the much like to Painlevé coordinate system. Hence, the analysis present here is more general.

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